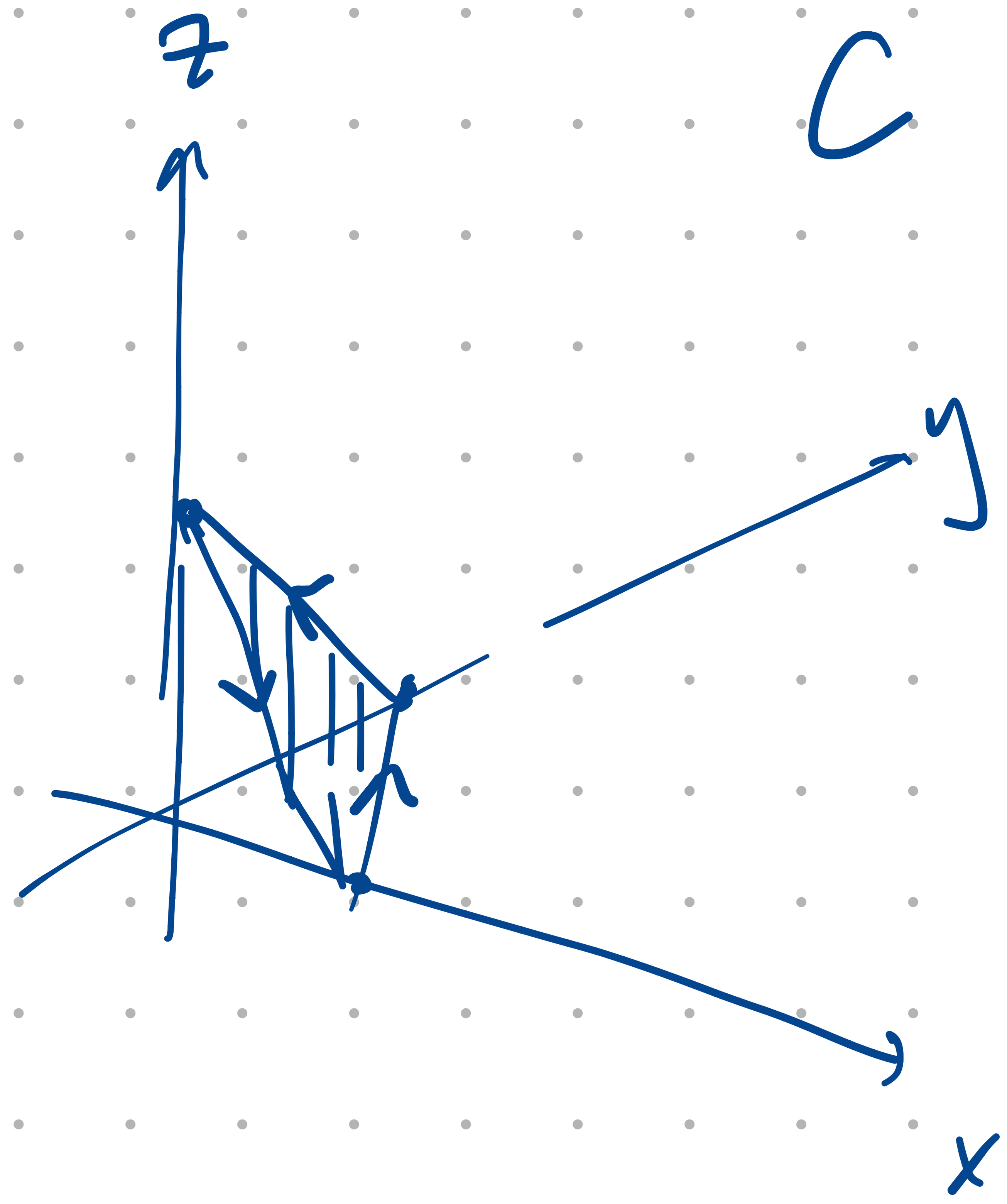
16.8 #7



C is "CCW when viewed from above"

In general: use RHR to figure out correct orientation.

But in these special cases:

surface		bounding curve
up ↑ (i.e. normal has + z component)	"positively oriented" (in the z dir)	CCW when viewed from above
down ↓ (normal has − z-component)	"negatively oriented" (in the z dir)	CW when viewed from above

$$\iint_{\text{surface}} \vec{F} \cdot d\vec{S}$$

"orientation" is about this and has nothing to do with $\vec{F}$.

Quiz 10 Summary:

#1) Is it possible to have

$$\nabla \times \vec{F} = \langle y, -x, z \rangle$$

for some $\vec{F}$?

two important identities: $\nabla \times \nabla f = \vec{0}$ always

$\nabla \cdot (\nabla \times \vec{F}) = 0$ always.

Sol:

$$0 = \nabla \cdot (\nabla \times \vec{F}) = \nabla \cdot \langle y, -x, z \rangle = 1$$

This is a contradiction, so such an $\vec{F}$ cannot exist. $\square$

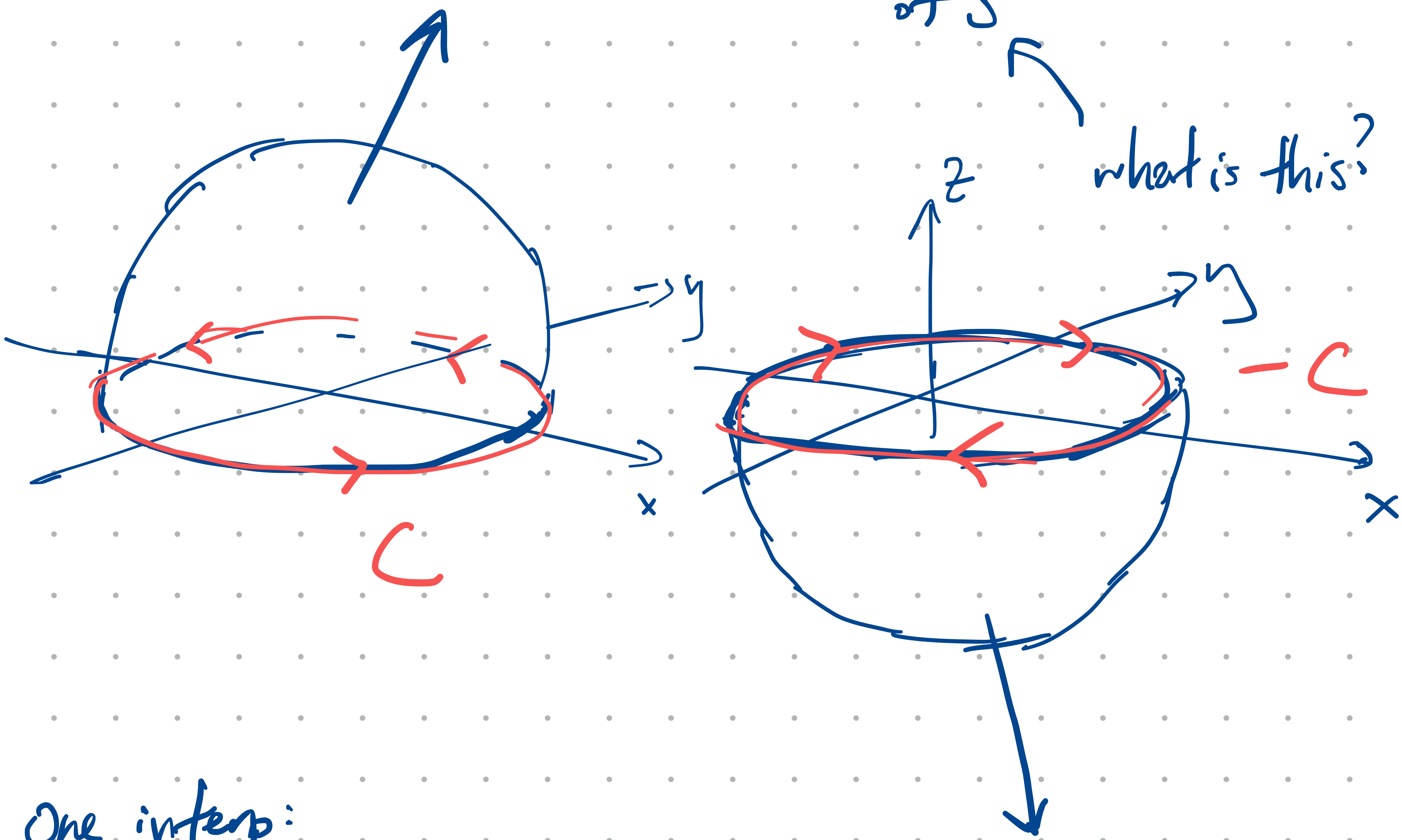
#2) S: $x^2+y^2+z^2=1$, outwards

$$\iint_S (\nabla \times \langle -y, x, z\rangle)\cdot d\vec{S}.$$

Method 1: Stokes

$$\iint_S (\nabla \times \langle -y, x, z\rangle)\cdot d\vec{S} = \int_{\substack{\text{boundary}\\ \text{of } S}} \langle -y, x, z\rangle \cdot d\vec{r}.$$

what is this?



One interp:

$$\iint_S (\nabla \times \langle -y, x, z\rangle)\cdot d\vec{S}$$

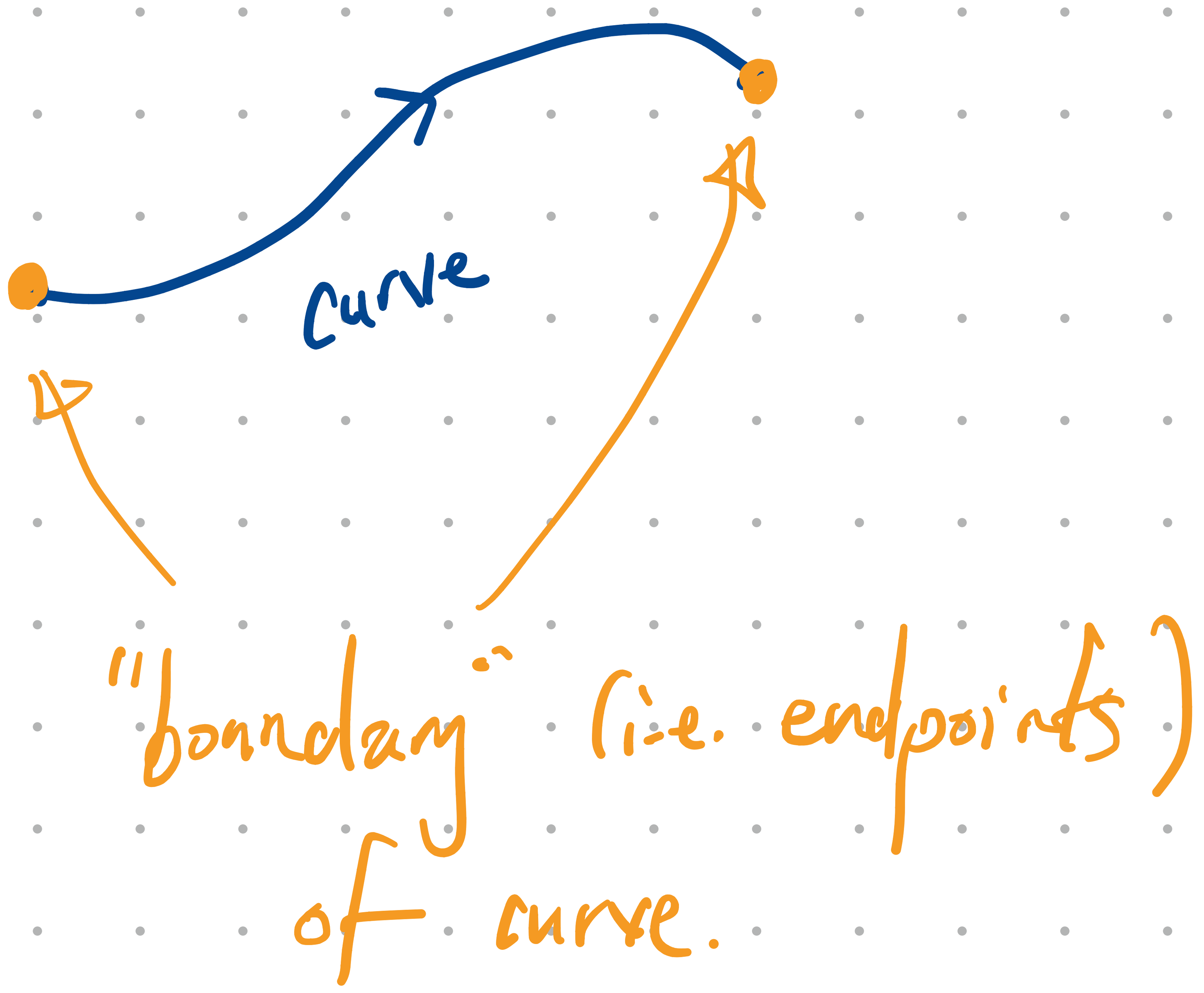
$$= \iint_{\substack{\text{upper}\\\text{hemi}}} (\nabla \times \langle -y, x, z\rangle) \cdot d\vec{S} + \iint_{\substack{\text{lower}\\\text{hemi}}} (\nabla \times \langle -y, x, z\rangle) \cdot d\vec{S}$$

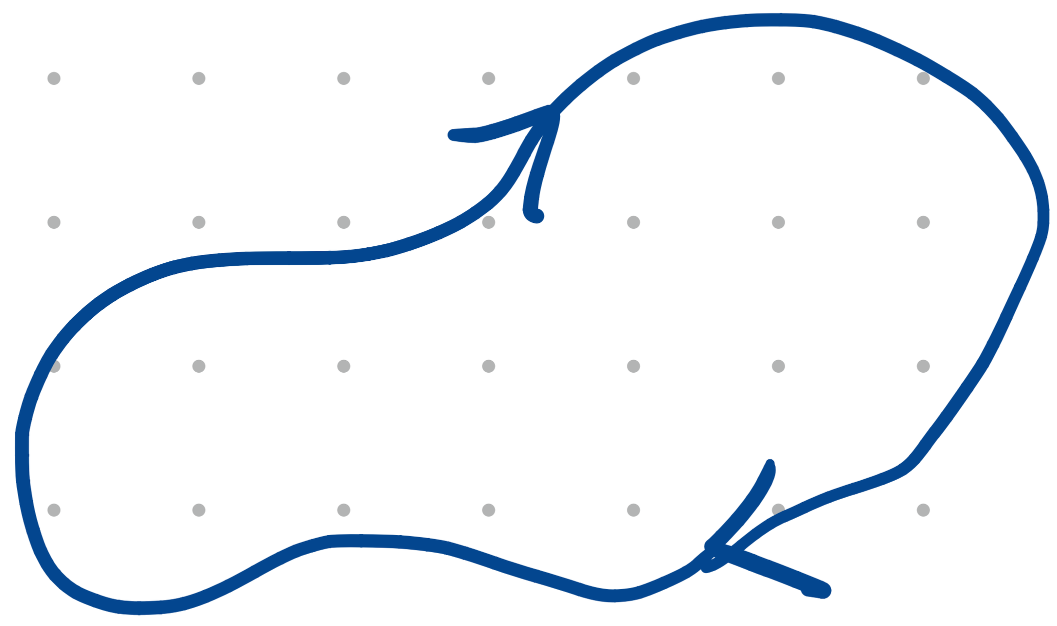
$$= \oint_{C} \langle -y, x, z\rangle \cdot d\vec{r} + \oint_{-C} \langle -y, x, z\rangle \cdot d\vec{r}$$

$$= \oint_{C} \langle -y, x, z\rangle \cdot d\vec{r} - \oint_{C} \langle -y, x, z\rangle \cdot d\vec{r} = 0.$$

<u>Another interpretation:</u>

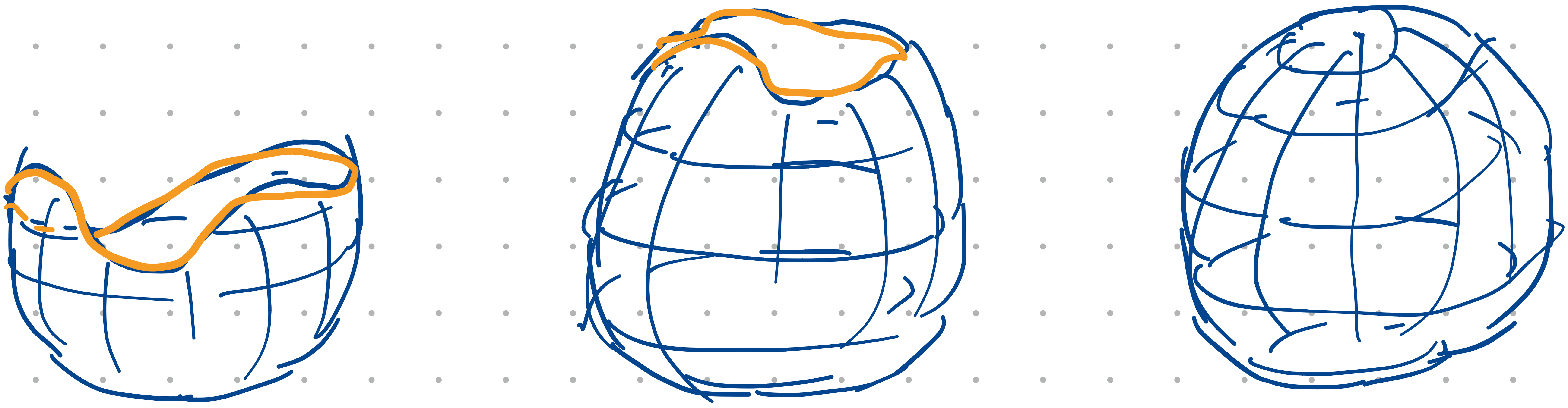
First let's consider a diff question...





what's the boundary of this curve?

it has no boundary.



there is no "edge" to this surface.

So Stokes just gives

$$\iint_S (\nabla \times \langle -y, x, z\rangle) \cdot d\vec{S} = \int_{\text{nothing}} \langle -y, x, z\rangle \cdot d\vec{r}$$

$= 0.$

Method 2: Divergence Thm

$$\oiint_S (\nabla \times \langle -y, x, z\rangle) \cdot d\vec{S} = \iiint_{\text{region enclosed by } S} \nabla \cdot (\nabla \times \langle -y, x, z\rangle)\, dV$$

closed surface, oriented outwards.

$$= \iiint_{x^2+y^2+z^2 \leq 1} 0 \;\; dV$$

If the problem had instead been

$$\iint_S \left(\nabla \times \left(\frac{\langle -y, x, z\rangle}{x^2+y^2+z^2}\right)\right)\cdot d\vec{S}$$

then the Stokes method would still work and conclude the integral is 0, but the divergence thm. <u>no longer works</u>.